Elliptic Ruijsenaars-Schneider model from the cotangent bundle over the two-dimensional current group.

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Abstract

It is shown that the elliptic Ruijsenaars-Schneider model can be obtained from the cotangent bundle over the two-dimensional current group by means of the Hamiltonian reduction procedure.

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1 Introduction

In our recent paper [1] we have shown that the elliptic Ruijsenaars-Schneider (RS) model [2] can be obtained by means of the Poisson reduction technique from the affine Heisenberg double. The aim of the present note is to derive the same RS model from the cotangent bundle over the two-dimensional centrally extended current group $GL(N)(z,\bar{z})$ applying this time the Hamiltonian reduction procedure [3]-[8]. It is worthwhile to note that the cotangent bundle over the centrally extended sl current algebra was used in [9, 10] to obtain the elliptic Calogero-Moser model. In this short note we shall not discuss the state of affairs in the problem, see [1] and references therein.

The plan of the paper is as follows. In the second section we briefly describe an infinite-dimensional phase space, which can be regarded as the cotangent bundle over the two-dimensional centrally extended current group. Then we fix the momentum map, corresponding to the natural action of the group and characterize the reduced phase space. The resulting L-operator appears to be equivalent to the L-operator of the RS model. In the third section we calculate the Poisson bracket of the reduced phase space variables and prove that it coincides with the one of the RS model. The reduction procedure leads to the dynamical r-matrix which just as in our previous paper is equivalent to the one obtained in [11].

2 Cotangent bundle over $GL(N)(z,\bar{z})$

The cotangent bundle $\mathcal{T}$ over the two-dimensional centrally extended current group $GL(N)(z,\bar{z})$ is a straightforward generalization of the cotangent bundle over affine $\widehat{GL(N)}$. The Poisson structure on $\mathcal{T}$ is defined as follows. Let $A(x,y) = \sum A_{mn}e^{imx+iny}$ and $g(x,y) = \sum g_{mn}e^{imx+iny}$ be formal Fourier series in variables x and y with values in $gl(N)$ and $GL(N)$ respectively. It is convenient to use variables $z = x + \frac{\Delta}{2\pi}y$ and $\bar{z} = x + \frac{\bar{\Delta}}{2\pi}y$, where Δ is a modular parameter with $\mathrm{Im}\Delta > 0$. In what follows we shall often use the notation $A(x,y) \equiv A(z)$ and $g(x,y) \equiv g(z)$. The matrix elements A_{mn} and g_{mn} can be regarded as generators of the algebra of functions on $\mathcal{T}$. In close analogy with $T^*\widehat{GL(N)}$ the Poisson structure can be written as

$$\begin{aligned}
\{A_1(z), A_2(w)\} &= \frac{1}{2}[P, A_1(z) - A_2(w)]\delta(z-w) - kP\frac{\partial}{\partial\bar{z}}\delta(z-w) \\
\{g_1(z), g_2(w)\} &= 0 \\
\{A_1(z), g_2(w)\} &= g_2(w)P\delta(z-w)
\end{aligned} \tag{2.1}$$

where k is a (fixed) central charge and $\delta(z)$ is the two-dimensional δ-function. Here we use a standard tensor notation and P is the permutation operator.

The action of $GL(N)(z,\bar{z})$ on $\mathcal{T}$

$$\begin{aligned}
A(z) &\rightarrow T^{-1}(z)A(z)T(z) + kT^{-1}(z)\bar{\partial}T(z), \\
g(z) &\rightarrow T^{-1}(z)g(z)T(z)
\end{aligned}$$

is Hamiltonian. Thereby, we can consider the Hamiltonian reduction of $\mathcal{T}$ over the action of $GL(N)(z,\bar{z})$.

The momentum map taking value in $gl(N)(z,\bar z)^*$ looks as follows:

$$M(z) = k\bar\partial g(z)g^{-1}(z) + A(z) - g(z)A(z)g^{-1}(z).$$

It is easy to check that $M(z)$ does generate the action of the current group. We fix the value of $M(z)$ as:

$$M(z) = -\frac{k}{2\mathrm{Im}\Delta}h + 2\pi ik\delta_\varepsilon(z)\frac{1-e^{-ix}}{i}K. \tag{2.2}$$

Here h and ε are arbitrary complex numbers,

$$\delta_\varepsilon(z) = \frac{2\pi}{\mathrm{Im}\Delta}\delta_\varepsilon(x)\delta(y),$$

$$\delta_\varepsilon(x) = \frac{1}{\varepsilon}\left(\theta(x+\frac{\varepsilon}{2}) - \theta(x-\frac{\varepsilon}{2})\right) = \frac{1}{2\pi i\varepsilon}\sum_{n=-\infty}^{n=+\infty}\frac{1}{n}(e^{in\frac{\varepsilon}{2}} - e^{-in\frac{\varepsilon}{2}})e^{inx},$$

and K is a constant matrix $K = e\otimes e^t$, where e is the N-dimensional vector with entries $e_i = 1/\sqrt{N}$.

To obtain a finite dimensional reduced phase space one has to consider the limit when ε goes to zero. To treat this limit we employ the same strategy as in [1]. We multiply the both sides of (2.2) by $g(z)$, that gives

$$k\bar\partial g(z) + A(z)g(z) - g(z)A(z) + \frac{k}{2\mathrm{Im}\Delta}hg(z) = 2\pi ik\delta_\varepsilon(z)K\frac{1-e^{-ix}}{i}g(z). \tag{2.3}$$

The l.h.s. of this equation does not have any explicit dependence on ε. As to the r.h.s., when ε tends to zero, $\delta_\varepsilon(z)$ becomes proportional to $\delta(x)\delta(y)$ and the r.h.s. is well defined only if the function $\frac{1-e^{-ix}}{i}g(x,0)$ is well defined at $x=0$. In this case $\lim_{\varepsilon\to 0}\delta_\varepsilon(z)\frac{1-e^{-ix}}{i}g(z) = \delta(z)Z$, where $Z = \frac{1-e^{-ix}}{i}g(x,0)|_{x=0}$.

So we define the constraint surface as being the solution of the equation

$$k\bar\partial g(z) + A(z)g(z) - g(z)A(z) + \frac{k}{2\mathrm{Im}\Delta}hg(z) = 2\pi ik\delta(z)KZ. \tag{2.4}$$

and in the following we shall explore solutions of this equation.

We start with the following differential equation

$$k\bar\partial g(z) + Dg(z) - g(z)D + \frac{k}{2\mathrm{Im}\Delta}hg(z) = 2\pi ik\delta(z)Y, \tag{2.5}$$

where D is a constant diagonal matrix and Y is an arbitrary constant matrix.
It is useful to introduce a function of two complex variables

$$W(z,s) = \frac{\sigma(z+s)}{\sigma(z)\sigma(s)}e^{-\frac{\zeta(\pi)}{\pi}zs}e^{is\frac{z-\bar z}{\Delta-\bar\Delta}}. \tag{2.6}$$

Here $\sigma(z)$ and $\zeta(z)$ are the Weierstrass σ- and ζ-functions with periods equal to 2π and Δ. This function is the only doubly periodic solution of the following equation:

$$\bar\partial W(z,s) + i\frac{s}{\Delta-\bar\Delta}W(z,s) = 2\pi i\delta(z) \tag{2.7}$$

In terms of W eq. (2.5) can be solved as

$$g(z)=\sum_{ij}\frac{\sigma(z+s_{ij})}{\sigma(z)\sigma(s_{ij})}e^{-\frac{\zeta(\pi)}{\pi}zs_{ij}}e^{is_{ij}\frac{z-\bar{z}}{\Delta-\bar{\Delta}}}Y_{ij}E_{ij}=\sum_{ij}W(z,s_{ij})Y_{ij}E_{ij}. \tag{2.8}$$

Here we introduce the notation $s_{ij}=q_i-q_j+h$, $q_i=\frac{\Delta-\bar{\Delta}}{ik}D_i$

Now we turn to the momentum map equation (2.4). By using a generic gauge transformation one can diagonalize the field A. Then equation (2.4) takes the form of eq.(2.5)

$$k\bar{\partial}g'(z)+Dg'(z)-g'(z)D+\frac{k}{2\mathrm{Im}\Delta}hg'(z)=2\pi ik\delta(z)K'Z', \tag{2.9}$$

where

$$A(z)=T(z)DT^{-1}(z)-k\bar{\partial}T(z)T^{-1}(z),\quad g(z)=T(z)g'(z)T^{-1}(z)$$

for some T and $Z'=xg'(x,0)|_{x=0}$. We also have

$$K'=T^{-1}(0)KT(0)=T^{-1}(0)e\otimes e^tT(0)=f\otimes v^t,\qquad <f,v>=1$$

i.e. $f=T^{-1}(0)e$ and $e^tT(0)=v^t$. According to (2.8) we find

$$g'(z)=\sum_{ij}W(z,s_{ij})(K'Z')_{ij}E_{ij}.$$

Taking the value of $xg'(x,0)$ at the point $x=0$ we arrive at the compatibility condition

$$Z'=K'Z'=f\otimes v^tZ',\qquad <f,v>=1.$$

The solution of this equation is $Z'=f\otimes c^t$, where c is an arbitrary vector. Now it is easy to find Z:

$$Z=T(0)Z'T^{-1}(0)=T(0)f\otimes c^tT^{-1}(0)=e\otimes c^tT^{-1}(0)\equiv e\otimes b^t$$

Thus, we get

$$k\bar{\partial}g(z)+A(z)g(z)-g(z)A(z)+\frac{k}{2\mathrm{Im}\Delta}hg(z)=2\pi ik(e\otimes e^t)(e\otimes b^t)\delta(z). \tag{2.10}$$

To summarize, eq.(2.4) has a solution for any field A and for any field g, such that $xg(x,0)|_{x=0}$ is of the form $e\otimes b^t$. For a fixed field A and a vector b this solution is unique. Note that, in general, $<b,e>\neq 1$. The form of the r.h.s. of (2.10) shows that the isotropy group of this equation is

$$G_{isot}=\{T(z)\subset G(z,\bar{z})\ \mid\ T(0)e=\lambda e,\lambda\in\mathbf{C}\}.$$

This group transforms a solution of (2.10) into another one, so the reduced phase space is defined as

$$\mathcal{P}_{red}=\frac{\text{all solutions of (2.4)}}{G_{isot}}.$$

The group G_{isot} is large enough to diagonalize the field A and hence we can parametrize the reduced phase space by the section (D, L), where L is a solution of (2.4) with $A = D$. One can easily see that $\mathcal{P}_{red}$ is finite dimensional and it's dimension is equal to $2N$, i.e. N coordinates of D plus N coordinates of the vector b. Due to eq.(2.8) the corresponding L-operator has the following form:

$$L(z) = \sum_{ij} \frac{\sigma(z+s_{ij})}{\sigma(z)\sigma(s_{ij})} e^{-\frac{\zeta(\pi)}{\pi} z s_{ij}} e^{i s_{ij} \frac{z-\bar{z}}{\Delta-\bar{\Delta}}} e_i b_j E_{ij} = \sum_{ij} W(z, s_{ij}) Y_{ij} E_{ij}. \tag{2.11}$$

$L(z)$ and the L-operator obtained in [1] are related by the gauge transformation with the diagonal matrix $e^{-i\frac{z-\bar{z}}{\Delta-\bar{\Delta}}q}$ and the shift of the spectral parameter: $z \to z - \frac{\Delta}{2}$.

The standard L-operator of the elliptic Ruijsenaars-Schneider model can be obtained by multiplying $L(z)$ by the function $\frac{\sigma(z)\sigma(h)}{\sigma(z+h)} e^{\frac{\zeta(\pi)}{\pi} zh - ih\frac{z-\bar{z}}{\Delta-\bar{\Delta}}}$ and performing the gauge transformation by means of the diagonal matrix $e^{(\frac{\zeta(\pi)}{\pi} z - i\frac{z-\bar{z}}{\Delta-\bar{\Delta}})q}$:

$$L^{Ruij}(z) = \frac{\sigma(z)\sigma(h)}{\sigma(z+h)} e^{\frac{\zeta(\pi)}{\pi} zh - ih\frac{z-\bar{z}}{\Delta-\bar{\Delta}}} e^{(\frac{\zeta(\pi)}{\pi} z - i\frac{z-\bar{z}}{\Delta-\bar{\Delta}})q} L(z) e^{-(\frac{\zeta(\pi)}{\pi} z - i\frac{z-\bar{z}}{\Delta-\bar{\Delta}})q} \tag{2.12}$$

3 The Poisson structure on the reduced space

Our goal in this section is to examine the Poisson structure on the reduced phase space. We should calculate the Poisson brackets for the coordinates D-s and b-s. Following the general Dirac procedure one should find a G_{isot}-invariant extension of functions on the reduced phase space $\mathcal{P}_{red}$ to a vicinity of $\mathcal{P}_{red}$ and then calculate the Dirac bracket.

The bracket for the coordinates D_i and b_i can be extracted from the bracket for D and $L(z)$. The simplest gauge invariant extension for D and $L(z)$ looks as follows:

$$D \to D[A] = T^{-1}(z)A(z)T(z) + kT^{-1}(z)\bar{\partial}T(z), \tag{3.13}$$

$$L(z) \to \mathcal{L}[A, g](z) = T^{-1}(z)g(z)T(z) \tag{3.14}$$

Some comments are in order. Eq.(3.13) is a solution of the factorization problem for $A(z)$. Generally this solution is not unique but we fix the matrix $T[A]$ by the boundary condition $T[A](0)e = e$ that kills the ambiguity up to the action of the Weil group (see, e.g. [10]) and makes (3.13) to be correctly defined. It is obvious that on $\mathcal{P}_{red}$: $T[A] = 1$ and $\mathcal{L}[A, g](z) = L(z)$.

The most interesting is the bracket for $\mathcal{L}(z)$ and $\mathcal{L}(w)$ defined by eq.(3.14). At the end of the section we shall comment on the contribution from the second class constraints to the Dirac bracket. By definition, one has

$$\begin{aligned} \{\mathcal{L}_1, \mathcal{L}_2\}_{\mathcal{P}_{red}} &= (\{T_1, T_2\}L_1L_2 - L_2\{T_1, T_2\}L_1 - L_1\{T_1, T_2\}L_2 \\ &+ L_1L_2\{T_1, T_2\} - \{T_1, g_2\}L_1 - \{g_1, T_2\}L_2 \\ &+ L_2\{g_1, T_2\} + L_1\{T_1, g_2\}\ )|_{\mathcal{P}_{red}} \end{aligned} \tag{3.15}$$

Here we took into account that $T[A]|_{\mathcal{P}_{red}} = 1$.

Let us first calculate

$$\{g_{ij}(z), T_{kl}(w)\} = \sum_{m,n} \int d^2z' \{g_{ij}(z), A_{mn}(z')\} \frac{\delta T_{kl}(w)}{\delta A_{mn}(z')}.$$

Performing the variation of the both sides of (3.13), we get

$$X(z) = t(z)D - Dt(z) - k\bar{\partial}t(z) + d, \tag{3.16}$$

where $X(z) = \delta A(z)$, $t(z) = \delta T(z)$ and $d = \delta D$.
The general solution of (3.16) is

$$t(z) = Q - \frac{1}{2\pi ik} \sum_{i,j} \int d^2w W(z - w, q_{ij}) X_{ij}(w) E_{ij}. \tag{3.17}$$

Here Q is some constant diagonal matrix and we introduce the following notation

$$w(z, 0) = \lim_{\varepsilon \to 0} (w(z, \varepsilon) - \frac{1}{s}) = \zeta(z) - \frac{\zeta(\pi)}{\pi} z + i \frac{z - \bar{z}}{\Delta - \overline{\Delta}}.$$

This function solves the equation

$$\bar{\partial} W(z, 0) = 2\pi i \delta(z) - \frac{i}{\Delta - \overline{\Delta}}$$

The solution $t(z)$ obeying the condition $t(0)e = 0$ has the following form

$$t(z) = \frac{1}{2\pi ik} \sum_{i,j} \int d^2w (W(-w, q_{ij}) X_{ij}(w) E_{ii} - W(z - w, q_{ij}) X_{ij}(w) E_{ij}) \tag{3.18}$$

Performing the variation of eq.(3.18) with respect to $X_{mn}(w)$ one gets

$$\frac{\delta T_{kl}(z)}{\delta A_{mn}(w)}|_{\mathcal{P}_{red}} \equiv Q^{kl}_{mn}(z, w) = \frac{1}{2\pi ik} \left(W(-w, q_{kn}) \delta_{kl} \delta_{km} - W(z - w, q_{kl}) \delta_{km} \delta_{ln} \right)$$

By using the Poisson bracket (2.1) we get

$$\{g_{ij}(z), T_{kl}(w)\}|_{red} = -L_{in}(z) Q^{kl}_{jn}(w, z).$$

Substituting $\{g, T\}$ and $\{T, g\}$ brackets into (3.15) we can rewrite the $\{\mathcal{L}, \mathcal{L}\}$ bracket in the following form:

$$\begin{aligned} \{\mathcal{L}_1(z), \mathcal{L}_2(w)\}|_{red} &= -L_1(z)L_2(w)k^+(z, w) - k^-(z, w)L_1(z)L_2(w) \\ &+ L_1(z)s^-(z, w)L_2(w) + L_2(w)s^+(z, w)L_1(z), \end{aligned} \tag{3.19}$$

where

$$\begin{aligned} k^-(z, w) &= -\{T_1(z), T_2(w)\}, \\ k^+(z, w) &= \omega(z, w) - P\omega(w, z)P - \{T_1(z), T_2(w)\}, \\ s^-(z, w) &= \omega(z, w) - \{T_1(z), T_2(w)\}, \\ s^+(z, w) &= -P\omega(w, z)P - \{T_1(z), T_2(w)\}, \end{aligned}$$

and $\omega_{ij\,kl}(z,w) = Q^{kl}_{ji}(w,z)$

It is easy to check that $Pk^{\pm}(z,w)P = -\delta(z-w) - k^{\pm}(w,z)$ and $Ps^{\pm}(z,w)P = \pm s^{\mp}(w,z)$. We also have one more important identity

$$k^{+}(z,w) + k^{-}(z,w) = s^{+}(z,w) + s^{-}(z,w).$$

To complete the calculation we need the bracket $\{T_{ij}(z), T_{kl}(w)\}$ on the reduced space. The straightforward manipulation leads to a divergent result. By this reason we define this bracket as follows:

$$\{T_{ij}(z), T_{kl}(w)\} = \frac{1}{2}\lim_{\varepsilon\to 0}\left(\{T_{ij}(z), T^{\varepsilon}_{kl}(w)\} + \{T^{\varepsilon}_{ij}(z), T_{kl}(w)\}\right),$$

where $T^{\varepsilon}_{kl}(z)$ is defined as a solution of the factorization problem with the boundary condition $T(\varepsilon)e = e$.

A simple calculation gives the following result for the bracket $\{T, T\}$

$$\begin{aligned} 2\pi ik\{T_{ij}(z), T_{kl}(w)\} = &\left((\zeta(q_{ik}) - \tfrac{\zeta(\pi)}{\pi}q_{ik})(1-\delta_{ik}) + (\zeta(z-w) + \zeta(w) - \zeta(z))\,\delta_{ik}\right)\delta_{ij}\delta_{kl} \\ + &\left(W(z-w, q_{ik})\delta_{il}\delta_{jk} + W(w, q_{ki})\delta_{il}\delta_{ij} - W(z, q_{ik})\delta_{jk}\delta_{kl}\right)(1-\delta_{ik}) \end{aligned}$$

By using this formula we get the following expression for the coefficients:

$$\begin{aligned} 2\pi ik\,k^{-}_{ij\ kl}(z,w) &= -\zeta(q_{ik})\delta_{ij}\delta_{kl}(1-\delta_{ik}) - (\zeta(z-w) + \zeta(w) - \zeta(z))\,\delta_{ij}\delta_{ik}\delta_{il} \\ &- \left(W(z-w, q_{ik})\delta_{il}\delta_{jk} + W(w, q_{ki})\delta_{il}\delta_{ij} - W(z, q_{ik})\delta_{jk}\delta_{kl}\right)(1-\delta_{ik}) \\ &+ \frac{\zeta(\pi)}{\pi}q_{ik}\delta_{ij}\delta_{kl} \\ 2\pi ik\,k^{+}_{ij\ kl}(z,y) &= \left(\zeta(z-w) - \frac{\zeta(\pi)}{\pi}(z-w) + i\frac{z-w-\bar{z}+\bar{w}}{\Delta-\overline{\Delta}}\right)\delta_{ij}\delta_{ik}\delta_{il} \\ &+ \left(W(z-w, q_{ik})\delta_{jk}\delta_{il} - \zeta(q_{ik})\delta_{ij}\delta_{kl}\right)(1-\delta_{ik}) \\ &+ \frac{\zeta(\pi)}{\pi}q_{ik}\delta_{ij}\delta_{kl} \\ 2\pi ik\,s^{-}_{ij\ kl}(z,w) &= -\left(\zeta(w) - \frac{\zeta(\pi)}{\pi}(w) + i\frac{w-\bar{w}}{\Delta-\overline{\Delta}}\right)\delta_{ij}\delta_{ik}\delta_{il} \\ &- \left(W(w, q_{ki})\delta_{ij}\delta_{il} + \zeta(q_{ik})\delta_{ij}\delta_{kl}\right)(1-\delta_{ik}) \\ &+ \frac{\zeta(\pi)}{\pi}q_{ik}\delta_{ij}\delta_{kl} \\ 2\pi ik\,s^{+}_{ij\ kl}(z,w) &= \left(\zeta(z) - \frac{\zeta(\pi)}{\pi}(z) + i\frac{z-\bar{z}}{\Delta-\overline{\Delta}}\right)\delta_{ij}\delta_{ik}\delta_{il} \\ &+ \left(W(z, q_{ik})\delta_{jk}\delta_{kl} - \zeta(q_{ik})\delta_{ij}\delta_{kl}\right)(1-\delta_{ik}) \\ &+ \frac{\zeta(\pi)}{\pi}q_{ik}\delta_{ij}\delta_{kl} \end{aligned}$$

One can easily verify that the last lines in the expressions obtained for k-s and s-s do not contribute to the bracket $\{\mathcal{L}, \mathcal{L}\}$.

To proceed further, let us note that (see eq.(2.11)):

$$L_{ii}(z) = \frac{1}{\sqrt{N}} W(z,h) b_i, \tag{3.20}$$

so the bracket $\{b_i, b_j\}$ follows from the $\{L_{ii}, L_{jj}\}$ bracket only. Just as in the case of the Heisenberg double one can check that the bracket of $\mathcal{L}_{ii}$ with the constraint (2.3) vanishes on $\mathcal{P}_{red}$ for any value of ε. Thus, there is no contribution from the Dirac term to the $\{L_{ii}, L_{jj}\}$ bracket.

Performing the same calculations as in [1] we arrive at

$$2\pi ik\{b_i, b_j\} = b_i b_j (2\zeta(q_{ij}) - \zeta(q_{ij}+h) - \zeta(q_{ij}-h)). \tag{3.21}$$

The bracket $\{\mathcal{L}, D\}$ and $\{D, D\}$ can be found by a similar device as was used above. The Dirac terms do not contribute as well. The final result reads

$$\{D[A]_1, D[A]_2\}|_{red} = 0, \tag{3.22}$$

$$2\pi(\Delta - \overline{\Delta})\{\mathcal{L}(z)_1, D[A]_2\}|_{red} = -\sum_{i,j} L_{ij}(z) E_{ij} \otimes E_{jj}. \tag{3.23}$$

Now for the reader's convenience we list the Poisson brackets obtained in terms of the coordinates on $\mathcal{P}_{red}$

$$\begin{aligned} \{q_i, q_j\} &= 0 \\ 2\pi ik\{q_i, b_j\} &= b_j \delta_{ij} \\ 2\pi ik\{b_i, b_j\} &= b_i b_j (2\zeta(q_{ij}) - \zeta(q_{ij}+h) - \zeta(q_{ij}-h)), \end{aligned} \tag{3.24}$$

this is just the Poisson structure of the elliptic Ruijsenaars-Schneider model.

Just as in [1] the bracket for the operator L^{Ruij} defined by eq.(2.12) being calculated by using (3.19) and (3.23) reproduces the bracket obtained in [11]. It means that there is no contribution from the Dirac term even for the nondiagonal matrix elements L_{ij}.

4 Conclusion

In this paper we have pointed out that the elliptic Ruijsenaars-Schneider model can be obtained by means of the Hamiltonian reduction procedure from the cotangent bundle over the two-dimensional current group. As compare to the scheme proposed in our previous paper [1] this one possesses a number of advantages. First of all in this scheme the calculations are drastically simplified. Then, it explains why the contribution from the trigonometric r-matrix which defines the Poisson structure on the Heisenberg double drops out from the final result.

It seems to be interesting to examine the Poissonian reduction of the Heisenberg double of the two-dimensional current group. In this case one could expect to obtain some generalization of the RS model.

It is known that the elliptic Calogero-Moser model is related to the Chern-Simons theory. Hence it is an interesting problem to find a field-theoretical formulation of the elliptic RS model.

ACKNOWLEDGMENT The authors are grateful to N.A.Slavnov for valuable discussions. This work is supported in part by the RFFR grants N96-01-00608 and N96-01-00551 and by the ISF grant a96-1516.

References

[1] Arutyunov G.E., Frolov S.A. and Medvedev P.B., Elliptic Ruijsenaars-Schneider model via the Poisson reduction of the Affine Heisenberg Double; hep-th/9607170.

[2] Ruijsenaars S.N., Comm.Math.Phys. 110 (1987) 191.

[3] V.I.Arnol'd, Mathematical methods of classical mechanics, Graduate Texts in Math., 60, Spinger-Verlag, Berlin, New-York, 1989.

[4] M.A.Olshanetsky, A.M.Perelomov, Invent. Math. 37 (1976) 93.

[5] Kazhdan D., Kostant B., Sternberg S., Comm.Pure Appl.Math. 31 (1978) 481.

[6] M.A.Olshanetsky, A.M.Perelomov, Phys. Reps. 71 (1981) 313.

[7] M.A.Olshanetsky, A.M.Perelomov, Phys. Reps. 94 (1983) 6.

[8] Avan J. and Talon M., Phys.Lett.B303 (1993) 33-37.

[9] Gorsky A. and Nekrasov N., Elliptic Calogero-Moser system from two-dimensional current algebra; Preprint hep-th/9401021.

[10] Arutyunov G.E. and Medvedev P.B., Generating equation for r-matrices related to dynamical systems of Calogero type; Preprint hep-th/9511070, to appear in Phys.Lett.A.

[11] Suris Yu.B., Why are the Ruijsenaars-Schneider and the Calogero-Moser hierarchies governed by the same r-matrix;